

On a Gyro-Thermal Effect with Polyatomic Gases in a Magnetic Field *

L. WALDMANN

Institut für Theoretische Physik der Universität Erlangen-Nürnberg, Erlangen

(Z. Naturforschg. 22 a, 1678—1681 [1967]; received 26 June 1967)

SCOTT, STURNER and WILLIAMSON observed that a polyatomic gas exerts a torque on a heated cylinder if a homogeneous magnetic field parallel to the axis is present. This effect is explained, for high enough pressure, by the kinetic theory of polyatomic gases. Decisive is the huge “amplification of spin” occurring when spin is transferred from molecules to cylinder in “cut tennis ball reflexion”. Phenomenologically this is manifested in the boundary condition (4): the gas shall have a slip velocity, tangential to the wall, which is proportional to the tangential component of its “azimuthal polarization”.

SCOTT, STURNER and WILLIAMSON¹, conducting measurements of the EINSTEIN-DE HAAS effect, observed an interesting “gyro-thermal” effect. A circular bronze cylinder (radius $R_i = 1$ cm, length $L = 20$ cm) is immersed as a delicate torsional pendulum in a polyatomic gas (O_2 , N_2 , CH_4 etc.) at pressures p_0 from 0.025 to 1 torr. The container is a coaxial glass cylinder of radius $R_a = 3.8$ cm. By heating, the inner cylinder is held at a temperature $T_i = T_a + \Delta T$, where T_a is the temperature of the container. The difference varied in the interval $0 < \Delta T < 55^\circ K$. A homogeneous magnetic field parallel to the axis is applied, $0.2 < H < 200$ Oe. The combination of heat flux and magnetic field results in a torque on the inner cylinder about its axis. The torque maximizes at a certain H^* , which is proportional to the pressure and depends on the gas. The maximal torque is of order 10^{-2} erg. No effect is observed with noble gases. A theoretical explanation of the effect has not been developed in¹.

The torque is surprisingly large at first sight. Let us suppose that by some mechanism the intrinsic molecular angular momenta, the “spins”, were polarized parallel to the cylinder axis with a component $\hbar = 10^{-27}$ erg s per particle, say. The number density at $p_0 = 0.05$ torr is $n_0 = 2 \cdot 10^{15} \text{ cm}^{-3}$, the thermal velocity at temperature T_0 is taken as $c_0 = 5 \cdot 10^4 \text{ cm s}^{-1}$. If each molecule hitting the cylinder surface $F = 100 \text{ cm}^2$ would transfer nothing but its spin to the cylinder, a torque $\hbar n_0 c_0 F = 10^{-5}$ erg would result. This is much smaller than the experimental torque. So, the present assumption that

the molecular spins would simply add to give the spin of the cylinder, has little to do with the essential effect. To get out of the dilemma one has to observe that this assumption would be true only if each molecule were completely thermalized in a wall collision. In reality such a complete thermalisation will not take place and, in consequence of this, an “amplification of spin” occurs by a factor of the order $R_i/a_B \approx 10^8$, where a_B is an atomic length.

To see this, let us consider more in detail a molecule hitting the cylinder surface perpendicularly. The molecular spin shall again have a preferential direction parallel to the cylinder axis (be polarized). In general, the rebounding molecule will not leave the surface in the normal direction, but with a tangential component of linear momentum, transverse to the original polarization. This is well known from a spinning (cut) rough tennis ball falling vertically on the lawn. It is reflected obliquely. During reflexion a horizontal impulse is transferred to the earth with the radius R of the earth as the lever arm. Hence, the spin imparted to the earth (with an imaginary fixed bearing at the centre) will be larger than the spin of the ball of radius a by a factor R/a . This in principle explains the large torques observed. A very weak polarization of incident molecules is now sufficient. Again, such molecules are reflected in such a way as to circulate around the cylinder. Speaking phenomenologically: a boundary condition will require the gas velocity to have a tangential component transverse to the incident spin polarization. By viscosity (in the high pressure

* Part of a Seminar held at the International Summer Session in Science on “Fundamental Problems in Statistical Mechanics II”, Noordwijk aan Zee (Netherlands), June 29, 1967.

¹ G. G. SCOTT, H. W. STURNER, and R. M. WILLIAMSON, General Motors Research Publication 606, Nov. 11, 1966, Phys. Rev. 158, 117 [1967].



regime) the ensuing COUETTE-like velocity pattern causes the torque on the inner cylinder (and on the outer one likewise).

That the impinging molecules are spin-polarized parallel to the magnetic field is already well known from the kinetic theory for particles with spin or polyatomic molecules^{2,3}. Let us assume that in such a gas a flux is maintained proportional to some polar vector ∇a , gradient of temperature for heat conduction or of composition for diffusion. No magnetic field first. Then the spins in general are "azimuthally polarized". This means that a particle with velocity \mathbf{c} has a preferential direction $\nabla a \times \mathbf{c}$ of spin. With such a velocity-spin correlation, a local mean value of $\mathbf{c} \times \mathbf{s}$ exists. It is understood that \mathbf{s} denotes the angular momentum divided by \hbar . We put

$$\langle \mathbf{c} \times \mathbf{s} \rangle / c_0 \sqrt{S(S+1)} \equiv \mathbf{a},$$

where $c_0 = \sqrt{3kT_0/m}$ is the thermal velocity and S is the magnitude of spin ($= \frac{1}{2}$ for the electron). The

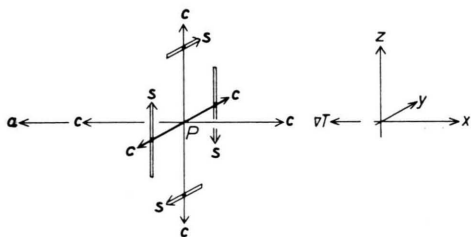


Fig. 1. Molecular velocity-spin distribution at point P with azimuthal polarization characterized by polar vector \mathbf{a} . Six orthogonal directions of velocities \mathbf{c} and accompanying spins \mathbf{s} indicated. No magnetic field, only temperature gradient ∇T . Coefficient l_{aq} of Eq. (1) chosen negative.

dimensionless polar vector \mathbf{a} (equivalent to $\mathbf{j}^{(2)}$ in² and $\mathbf{a}^{(2)}$ in⁴) adequately describes the gas being in this state of "azimuthal polarization", see Fig. 1. Without magnetic field the azimuthal polarization \mathbf{a} is parallel to the generating gradient ∇a . With a magnetic field however, \mathbf{a} also has a transverse component in the direction of $\mathbf{H} \times \nabla a$ ⁵, see Fig. 2. By the way, with this component is connected a transverse, HALL-effect-like heat or diffusion flux^{6,5}. Especially with a temperature gradient one has the azimuthal polarization, cf.⁵,

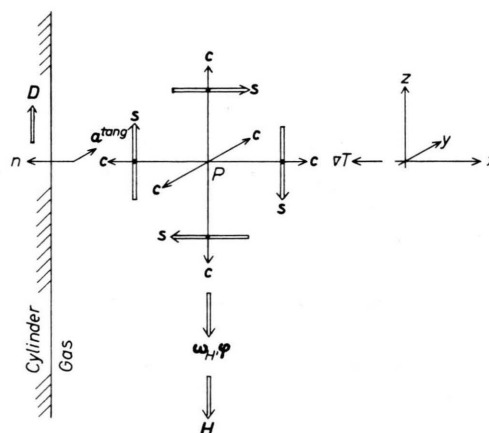


Fig. 2. Additional (transverse) azimuthal polarization \mathbf{a}^{tang} caused by magnetic field \mathbf{H} in nitrogen (negative magnetic moment), according to $\boldsymbol{\varphi} \times \nabla T$ term of Eq. (1) with $l_{aq} < 0$. Pertaining (additional) molecular velocity-spin distribution at point P indicated. Torque \mathbf{D} on cylinder in N_2 indicated at left. If $l_{aq} < 0$ were true, spinning N_2 molecules approaching cylinder in minus x direction would be reflected preferentially in minus y direction, as inferred from experimental $\gamma l_{aq} > 0$.

$$\mathbf{a} = - \sqrt{\frac{5}{27}} l_{aq} \left[\frac{1}{1+\varphi^2} \nabla + \frac{1}{1+\varphi^2} \boldsymbol{\varphi} \times \nabla + \frac{1-\alpha}{(1+\varphi^2)(1+\alpha\varphi^2)} \boldsymbol{\varphi} (\boldsymbol{\varphi} \cdot \nabla) \right] T/T_0. \quad (1)$$

By the way, the heat flux (equivalent to $\mathbf{a}^{(3)}$ in⁴) is, cf.⁵,

$$\mathbf{q} = - \frac{5}{6} p_0 c_0 \left[\left(l_{aq} + \frac{l_{aq}^2 \varphi^2}{l_{aa}(1+\varphi^2)} \right) \nabla - \frac{l_{aq}^2}{l_{aa}(1+\varphi^2)} \boldsymbol{\varphi} \times \nabla - \frac{l_{aq}^2 (1-\alpha)}{l_{aa}(1+\varphi^2)(1+\alpha\varphi^2)} \boldsymbol{\varphi} (\boldsymbol{\varphi} \cdot \nabla) \right] T/T_0.$$

The axial vector

$$\boldsymbol{\varphi} = l_{aa} \boldsymbol{\omega}_H / 2 c_0 \quad (2)$$

denotes half the angle of precession of the molecular magnetic moment $\mu \leq 0$ during the time of free

flight along the free path l_{aa} (termed $l_{-1}^{(22)}$ in⁴), the angular velocity of precession for spin S being

$$\boldsymbol{\omega}_H = -\mu \mathbf{H} / \hbar S. \quad (3)$$

The α is a ratio of free paths, presently immaterial.

² L. WALDMANN, Z. Naturforsch. **12a**, 660 [1957]; Nuovo Cimento **14**, 898 [1959]; Z. Naturforsch. **15a**, 19 [1960].

³ L. WALDMANN, Proc. Intern. Seminar on the Transport Properties of Gases, Brown University, Providence R.I. (1964), p. 59.

⁴ S. HESS and L. WALDMANN, Z. Naturforsch. **21a**, 1529 [1966].

⁵ L. WALDMANN and H. D. KUPATT, Z. Naturforsch. **18a**, 86 [1963].

⁶ J. KORVING, H. HULSMAN, H. F. P. KNAAP, and J. J. M. BEENAKKER, Phys. Letters **21**, 5 [1966].

The important cross coefficient $l_{aq} = -l_{qa} \leq 0$ (termed $l_{-1}^{(23)}$ in ⁴) has the dimension of a length too. It originates from that term of the binary scattering operator which is linear in the spin operator ². Such a term in general will exist for rotating molecules. By the way, l_{qq} (termed $l_{-1}^{(33)}$ in ⁴) is the free path for heat conduction.

What is now the fore-mentioned boundary condition for the gas velocity \mathbf{v} ? According to the meaning of \mathbf{a} , particles travelling in the direction $+\mathbf{n}$ (unit vector) normal towards the surface have a preferential spin $\langle \mathbf{s} \rangle_+ = \mathbf{a} \times \mathbf{n}$, see Fig. 2, and, in leaving the surface, will "roll aside" with a tangential component of velocity proportional to

$$c_0 \mathbf{n} \times \langle \mathbf{s} \rangle_+ = c_0 \mathbf{n} \times (\mathbf{a} \times \mathbf{n}) = c_0 \mathbf{a}^{\text{tang}},$$

the tangential component of the azimuthal polarization vector. Hence, as a phenomenological boundary condition one has to take (the factor $\sqrt{27/5}$ is inserted for convenience)

$$\mathbf{v}^{\text{tang}} = \gamma \mathbf{a}^{\text{tang}} \sqrt{27/5} \cdot c_0. \quad (4)$$

The numerical coefficient $\gamma \leq 0$, "the gyro velocity slip number", will depend on both interactions: particles with surface and with each other. If the particles were exactly thermalized in a wall collision, γ would vanish.

In the cylinder experiment¹, after (4) and (1) a tangential (circular) velocity is set up ($\varphi = \varphi_z$, < 0 in Fig. 2)

$$v_{i(a)} = -c_0 \frac{\varphi}{1+\varphi^2} \frac{\gamma l_{aq}}{T_0} \left(\frac{dT}{dr} \right)_{i(a)} \quad (5)$$

at the inner (outer) cylinder. The radius r , the positive direction of v and of the axis (upwards) form a right-handed coordinate system. The ensuing COUETTE flow pattern (hydrodynamical regime with pressure ≥ 0.15 torr) is

$$v = \frac{(v_i R_a - v_a R_i) r^{-1} - (v_i R_a^{-1} - v_a R_i^{-1}) r}{R_i^{-1} R_a - R_a^{-1} R_i}.$$

The torque on the inner cylinder in direction of the axis is (η = viscosity)

$$D_i = -D_a = -4\pi\eta \frac{v_i R_a - v_a R_i}{R_i^{-1} R_a - R_a^{-1} R_i} L.$$

⁷ If different values $\gamma_{i,a}$ are valid for both of the cylinders, the γ in (6) has to be replaced by

$$(\gamma_i R_i^{-1} R_a - \gamma_a R_a^{-1} R_i) / (R_i^{-1} R_a - R_a^{-1} R_i).$$

But this possibility is disregarded in the discussion. — As in (1), T_0 means some temperature between T_i and T_a .

The temperature gradients are

$$\left(\frac{dT}{dr} \right)_{i(a)} = (T_a - T_i) / R_{i(a)} \ln(R_a/R_i).$$

Inserting yields the torque on the inner cylinder⁷

$$D_i = -4\pi\eta c_0 \frac{\varphi}{1+\varphi^2} \gamma l_{aq} \frac{T_i - T_a}{T_0 \ln(R_a/R_i)} L. \quad (6)$$

This formula reasonably accounts for the experimental results¹ with pressures ≥ 0.15 torr.

1. With fixed pressure, $|D_i|$ goes through a maximum with increasing magnetic field. The maximum is attained for $|\varphi| = |\varphi^*| = 1$, i. e. after (2) for a precession frequency

$$|\omega_{H^*}| = 2c_0/l_{aa} \sim p_0.$$

Hence, H^* itself is proportional to the pressure p_0 , a fact which is familiar from the SENFTLEBEN effect.

2. The maximum value of $|D_i|$ is proportional to $|\gamma l_{aq}|$. The γ , as a number, is pressure independent; l_{aq} , like any free path, is inversely proportional to the pressure and so is $|D_i|_{\text{max}}$. This in deed is exhibited by the measurements, especially for O_2 and $p_0 \geq 0.15$ torr. (For lower pressure one is in the transition region and the hydrodynamical treatment breaks down.)

3. N_2 has a negative magnetic moment, cf. ⁸. Let us take the magnetic field to be downward. Then φ^* is -1 . Inserting $\eta = 1.7 \cdot 10^{-4}$ dyn cm⁻² s, $c_0 = 4.9 \cdot 10^4$ cm s⁻¹, the data of the arrangement ($T_a < T_i$) and $D_{i \text{ extr}} = +16 \cdot 10^{-3}$ erg, observed¹ with N_2 at 0.10 torr (150 Oe), one obtains

$$\gamma l_{aq} = 2.1 \cdot 10^{-4} \text{ cm.}$$

Introducing for comparison a free path $l = 0.044$ cm for N_2 at 0.10 torr one has as a possible combination, giving the observed γl_{aq} ,

$$\gamma = l_{aq}/l = \pm 0.07.$$

This is quite a reasonable order of magnitude for both. The gyrothermal effect is linear in l_{aq} (depends on its sign, but on that of γ too). The transverse heat flux is proportional to l_{aq}^2 . So, there seems to be no possibility to detect the signs of l_{aq} and γ separately. For this purpose one would have to carry

⁸ H. F. P. KNAAP and J. J. M. BEENAKKER, *Physica* **33**, 643 [1967].

out the molecular beam experiment mentioned previously³.

4. The velocities of the N₂ gas under the said conditions are $v_i = -0.40$, $v_a = -0.10$ cm s⁻¹. — That much about comparison with experiment.

The gyro-thermal motion of the gas will give rise to a pressure gradient under suitable circumstances. The gas shall be in a channel formed by the parallel container plates at $x = \pm d/2$, held at different temperatures, with magnetic field in z direction. At the plates the gas slips with the gyro-thermal velocity v_g in y direction. Stationarily, when no net transport of gas through the channel takes place, the gas flows back in y direction at $x=0$. The pertaining pressure gradient is

$$dp/dy = (12 \eta/d^2) v_g.$$

For N₂ at 0.10 torr, optimal magnetic field (in $-z$ direction, $\varphi^* = -1$), $d = 0.1$ cm and $T(-d/2) - T(d/2) = 30^\circ\text{K}$ one finds from (1) and (4) with

$$c_0 = 4.9 \cdot 10^4 \text{ cm s}^{-1}, \quad \gamma l_{aq} = 2.1 \cdot 10^{-4} \text{ cm}$$

⁹ *Note added in proof:* The experiments¹ showed however that the torque was not changed when the pendulum surface had been covered with Teflon tape. It is difficult to reconcile (4) with this observation. A theory of the effect, based on the internal properties of the gas only (pressure tensor also linked with second spatial derivatives of tempe-

for the slip velocity

$$v_g = -c_0 \frac{\varphi}{1+\varphi^2} \gamma l_{aq} \frac{1}{T_0} \frac{dT}{dx} = -5.1 \text{ cm s}^{-1}. \quad (7)$$

The ensuing pressure gradient with $\eta = 1.7 \cdot 10^{-4}$ dyn cm⁻² s is $dp/dy = -0.75 \cdot 10^{-3}$ torr cm⁻¹. With 20 cm channel length one has a pressure difference of 0.015 torr. It should be possible to detect it.

Eq. (4) in connection with (1) says that with a temperature gradient parallel to the wall there is a gyro slip velocity even in absence of a magnetic field. This will add to MAXWELL's thermal slip known for monatomic gases. With mixtures, different v_g -values might apply, leading to a separation of components.

The KNUDSEN regime has to be treated separately. This problem is not attacked here.

Hopefully, the gyro-thermal effect and similar phenomena, properly understood, will give new information about rotating molecules, especially about their interaction with surfaces⁹.

I thank Dr. S. HESS for discussions.

rature, "third" CHAPMAN-ENSKOG approximation), has been developed by LEVI and BEENAKKER, Phys. Letters, in print. In this theory the properties of the surface are not essential. The dependence of the torque on pressure and field is similar after both theories.